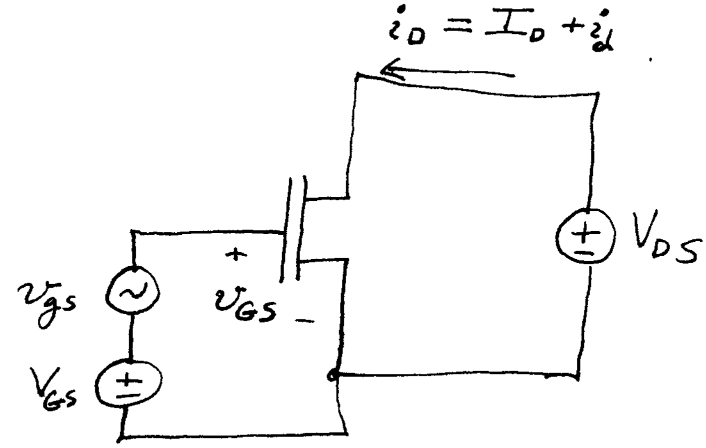
LOW FREQUENCY SMALL-SIGNAL (9.1.1)

I_D SATURATION



DC: V_{DS} (U.C.)$_{U.C}$

AC: v_{ds} (LC)$_{LC}$

TOTAL: v_{DS} (LC)$_{UC}$

DC quantity >> AC quantity ($V_{GS} >> v_{gs}$)
(small signal)

$$i_D = i_d + I_D = \frac{\beta}{2}\big(\underbrace{V_{GS} + v_{gs}}_{v_{GS}} - V_{THN}\big)^2 (1 + \lambda V_{DS})$$

A. TRANSCONDUCTANCE (g_m)

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{V_{GS}, I_D} = \beta (V_{GS} - V_{THN}) (1 + \lambda V_{DS})$$

(Evaluated at D.C. bias point @ Q = operating point)

$\Delta i_D = g_m \Delta v_{GS}$

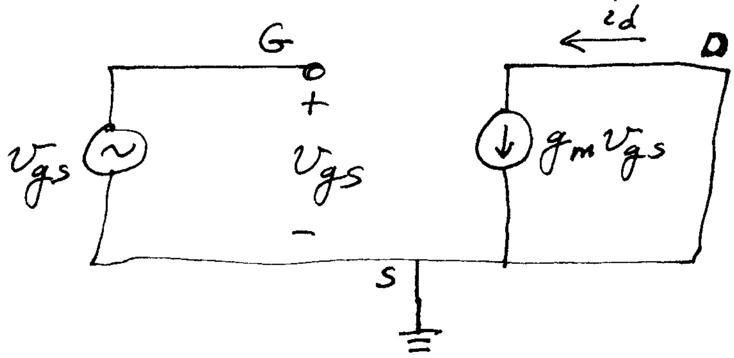
$\left.\begin{array}{l}\Delta i_D \equiv i_d \\ \Delta v_{GS} \equiv v_{gs}\end{array}\right\}$ A.C. quantities

SO $i_d = g_m v_{gs}$

FOR $\lambda V_{DS} \ll 1$,

$$g_m = \beta (V_{GS} - V_{THN}) = \sqrt{2\beta I_D}$$

LOW FREQ SMALL SIGNAL CIRCUIT



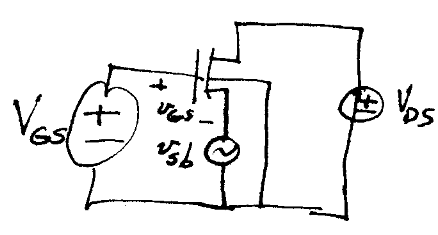
Small-signal model is a 1st order (linear) expansion of non-linear model.

FOR v_{gs} too large $\Rightarrow$ non-linear effects i.e. distortion, harmonics.

B. BODY TRANSCONDUCTANCE ($V_{SB} \neq 0$)

$$V_{TNN}(V_{SB}) = V_{THNO} + \gamma \left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right]$$

$$g_{mb} = - \left. \frac{\partial i_D}{\partial v_{sb}} \right|_{V_{SB}, I_D}$$



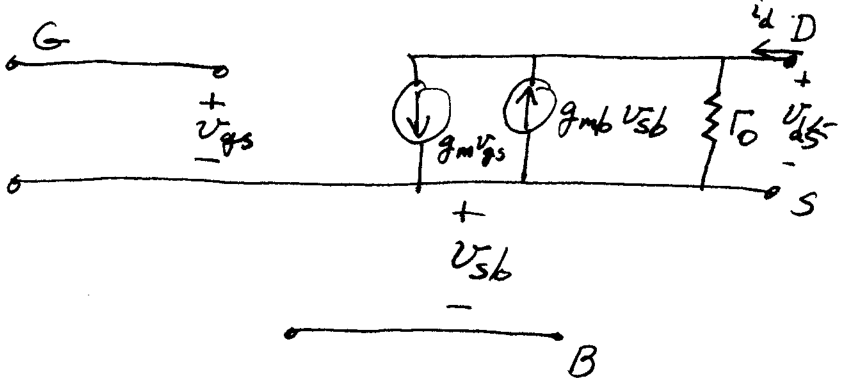
$$= \frac{-\partial i_d}{\partial V_{THN}} \frac{\partial V_{THN}}{\partial v_{sb}}$$

$v_{sb} = -v_{gs}$

$$= g_m \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$$

$$\equiv g_m \eta \qquad (0 \leq \eta \lesssim 0.6)$$

$$i_d = g_m v_{gs} - g_{mb} v_{sb}$$



C. Input Resistance $= \infty$ (Low f)

$$\left(\frac{\partial i_G}{\partial v_{gs}}\right)^{-1}$$

D. Output Resistance (r_o)

$$r_o^{-1} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{V_{DS},\, I_D} = \frac{\beta}{2}\left(V_{GS} - V_{THN}\right)^2 \lambda$$

$$\approx I_D \lambda \quad (\lambda V_{DS} << 1)$$

NOTE: λ often defined by an "Early Voltage" (V_A)

$$\lambda = \frac{1}{V_A}$$

$$r_o = \frac{1}{I_D \lambda} = \frac{V_A}{I_D}$$

E. VOLTAGE GAIN ($v_{sb} = 0$)

$$\frac{v_{ds}}{v_{gs}} = \frac{-g_m v_{gs} r_o}{v_{gs}} = -g_m r_o$$

II. TRIODE or LINEAR REGION

$I_D = \beta[(V_{GS}-V_{THN})V_{DS} - V_{DS}^2/2]$

Output RESISTANCE

$$R_{ch}^{-1} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{V_{DS}, I_D} = \beta[V_{GS} - V_{THN} - V_{DS}]$$

$$R_{CH} = \frac{1}{\beta(V_{GS} - V_{THN} - V_{DS})}$$

FOR $V_{DS} << V_{GS} - V_{THN}$

$$R_{CH} = \frac{1}{\beta(V_{GS} - V_{THN})} = \frac{1}{g_m}$$

III. SUBTHRESHOLD (WEAK INVERSION)

$$I_D = I_{DO}\frac{W}{L}e^{q(V_{GS}-V_{THN})/(NO \cdot k_B T)}$$

$$\cancel{\frac{\partial I_D}{\partial v_{gs}}}$$

$$i_D = I_{DO}\frac{W}{L}e^{q(v_{GS}-V_{THN})/(NO \cdot k_B T)}$$

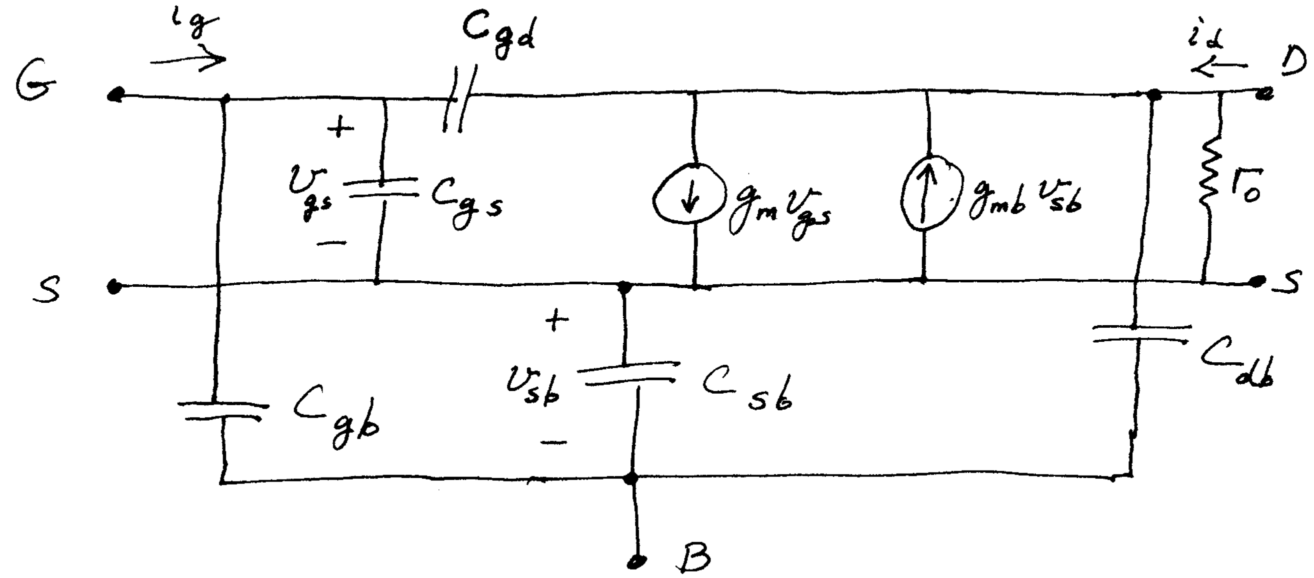
$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{V_{GS},} = \frac{I_D q}{NO \cdot k_B T}$$

(same form as for a diode)

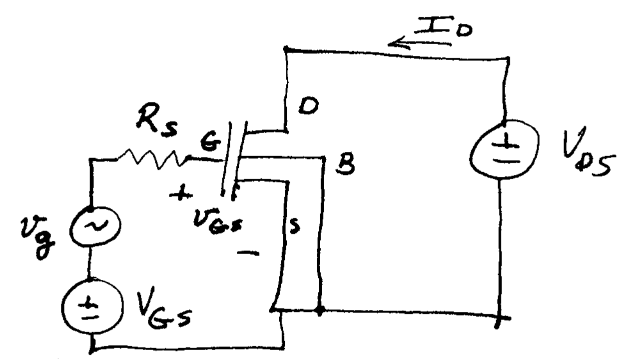
$$= \frac{I_D}{NO \cdot V_T}$$

$V_T = \frac{k_B T}{q} = 26\,mV$ @ 300K

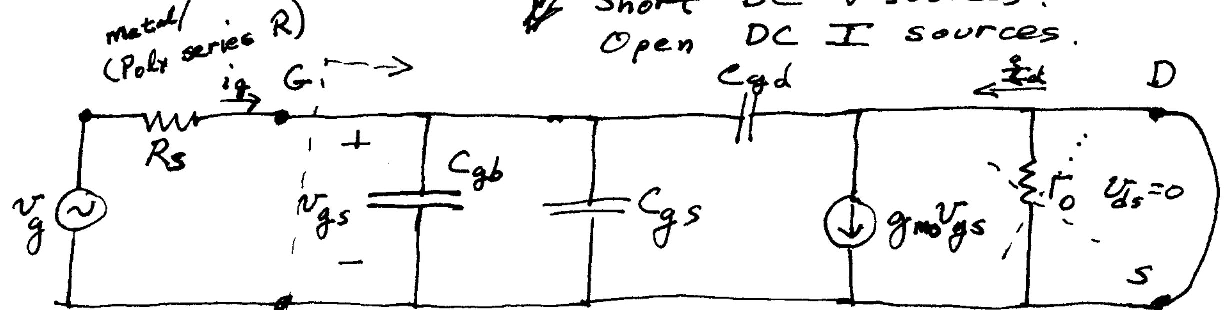
9.2 HIGH FREQUENCY SMALL SIGNAL

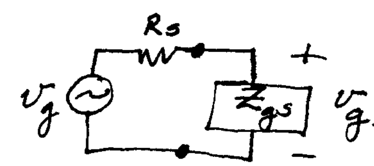


TRANSCONDUCTANCE $g_m(\omega)$



Short DC V sources.
Open DC I sources.





$$Z_{gs} = \frac{1}{j\omega(C_{gb} + C_{gs} + C_{gd})}$$

$$v_{gs} = v_g \frac{Z_{gs}}{R_S + Z_{gs}} = v_g \frac{\frac{1}{j\omega C_{TOT}}}{R_S + \frac{1}{j\omega C_{TOT}}}$$

$$v_{gs} = v_g \frac{1}{1 + j\omega R_s (C_{gb} + C_{gs} + C_{gd})}$$

$$\cancel{g_m(\omega) = \frac{i_d}{v_g}}$$

Assume that $|i_d| >> |v_{gs} j\omega C_{gd}|$

(feedthrough negligeble)

Then $i_d \cong g_{mo} v_{gs}$

$$g_m^{(\omega)} \equiv \frac{i_d}{v_g} = \frac{g_{mo}}{1 + j\omega R_s (C_{gb} + C_{gs} + C_{gd})}$$

FOR $R_s = 0$, $g_m(\omega) = g_{mo}$

g_{mo} = low freq. g_m.

<u>CUTOFF FREQUENCY</u>

The frequency when $\left|\frac{i_d}{i_g}\right| = 1$ (unity current gain)

AGAIN, Assuming

$$i_d \cong g_{mo} v_{gs} >> |j\omega C_{gd} v_{gs}|$$

(Not always a great assumption)

$$\left|\frac{i_d}{i_g}\right| = \left|\frac{g_{mo} \cancel{v_{gs}}}{j\omega_T (C_{gb} + C_{gs} + C_{gd}) \cancel{v_{gs}}}\right| = 1$$

$$\boxed{f_T = \frac{g_{mo}}{2\pi (C_{gb} + C_{gs} + C_{gd})}}$$

$$C_{gs} >> C_{gb}, C_{gd}$$

$$f_T \approx \frac{g_{mo}}{2\pi C_{gs}}$$

In saturation $C_{gs} = \frac{2}{3} W \cdot L \cdot C_{ox}'$ (Table 5.1)

$$g_{mo} \cong \mu_n C_{ox}' \frac{W}{L} (V_{GS} - V_{THN})$$

$$\boxed{f_T \approx \frac{3 \mu_n}{4\pi L^2} (V_{GS} - V_{THN})} \Rightarrow \text{minimum } L \text{ for max } f_T$$

9.3 Temperature Effects

$$\cancel{g_m(T}) \; I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{THN})^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{THN})^2$$

$$\left. \begin{array}{l} V_{THN}(T) \\ \mu_n(T) \end{array} \right\} \Rightarrow I_D(T), \; g_m(T)$$

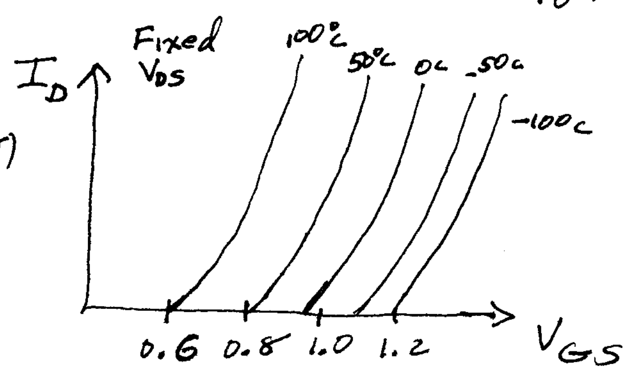
Temperature Coefficient of V_{THN}

$$TCV_{THN} = \frac{1}{V_{THN}} \frac{dV_{THN}}{dT} \approx -3000 \frac{ppm}{°C}$$

$$= \frac{-0.003}{°C}$$

$$\mu(T) = \mu(T_0) \left(\frac{T}{T_0} \right)^{-1.5}$$

$V_{TH}(T)$ Effect



$V_{THN} \uparrow$ as $T \downarrow$